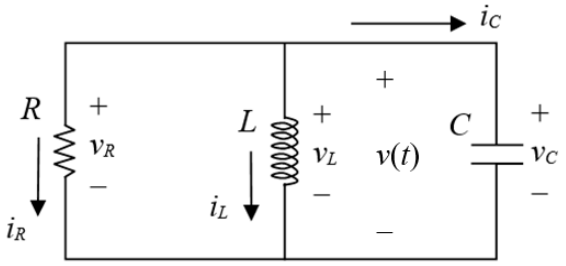
Chap 4 High Order Linear Circuits

4.1 Second-order RLC circuits without any sources

4.1.1 Parallel RLC circuit without any sources



- Component equations with initial conditions i_{L0} and v_{C0}:

(4.1.1-1) $\quad i_R(t)=\dfrac{v(t)}{R},\quad i_L(t)=\dfrac{1}{L}\int_0^t v(\tau)\,d\tau+i_{L0},\quad i_C(t)=Cv'(t)$

- Mathematic model

(4.1.1-2) $\quad v_R(t)=v_L(t)=v_C(t)=v(t)$

(4.1.1-3) $\quad i_R(t)+i_L(t)+i_C(t)=\dfrac{v(t)}{R}+\dfrac{1}{L}\int_0^t v(\tau)\,d\tau+i_{L0}+Cv'(t)=0$

$$\Rightarrow\ \frac{\hat{v}(s)}{R}+\frac{1}{sL}\hat{v}(s)+\frac{i_{L0}}{s}+sC\hat{v}(s)-Cv_{C0}=0$$

Hence, we have

(4.1.1-4) $\quad \hat{v}(s)=\dfrac{sv_{C0}-\dfrac{i_{L0}}{C}}{s^2+\dfrac{s}{RC}+\dfrac{1}{LC}}=\dfrac{sv_{C0}-\dfrac{i_{L0}}{C}}{s^2+2\alpha s+\omega_0^2}=\dfrac{sv_{C0}-\dfrac{i_{L0}}{C}}{s^2+2\xi\omega_0 s+\omega_0^2}$

where $\alpha=\dfrac{1}{2RC}$, $\omega_0=\dfrac{1}{\sqrt{LC}}$ and $\xi=\dfrac{\alpha}{\omega_0}=\dfrac{1}{2R}\sqrt{\dfrac{L}{C}}$.

- Characteristic roots λ_1 and λ_2

The characteristic equation is $s^2+2\xi\omega_0 s+\omega_0^2=(s-\lambda_1)(s-\lambda_2)$ and thus

(4.1.1-5) $\quad \hat{v}(s)=\dfrac{sv_{C0}-\dfrac{i_{L0}}{C}}{s^2+2\xi\omega_0 s+\omega_0^2}=\dfrac{sv_{C0}-\dfrac{i_{L0}}{C}}{(s-\lambda_1)(s-\lambda_2)}$

where $\lambda_1,\lambda_2=\left(-\xi\pm\sqrt{\xi^2-1}\right)\omega_0$ are the characteristic roots.

- [A] $\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} > 1$

  That means $\lambda_2 < \lambda_1 < 0$ and

  (4.1.1-6) $\hat{v}(s) = \frac{sv_{C0} - \frac{i_{L0}}{C}}{(s-\lambda_1)(s-\lambda_2)} = \frac{A_1}{s-\lambda_1} + \frac{A_2}{s-\lambda_2}$

  $\Rightarrow v(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

  where $A_1 = \frac{\lambda_1}{\lambda_1 - \lambda_2} v_{C0} - \frac{1}{\lambda_1 - \lambda_2}\frac{i_{L0}}{C}$ and $A_2 = -\frac{\lambda_2}{\lambda_1 - \lambda_2} v_{C0} + \frac{1}{\lambda_1 - \lambda_2}\frac{i_{L0}}{C}$.

- [B] $\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} = 1$

  That means $\lambda_2 = \lambda_1 = -\xi\omega_0 = -\alpha$ and

  (4.1.1-7) $\hat{v}(s) = \frac{sv_{C0} - \frac{i_{L0}}{C}}{(s+\alpha)^2} = \frac{B_1}{s+\alpha} + \frac{B_2}{(s+\alpha)^2}$

  $\Rightarrow v(t) = (B_1 + B_2 t)e^{-\alpha t}$

  where $B_1 = v_{C0}$ and $B_2 = -v_{C0}\alpha - \frac{i_{L0}}{C}$.

- [C] $\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} < 1$

  That means $\lambda_1, \lambda_2 = \left(-\xi \pm j\sqrt{1-\xi^2}\right)\omega_0 = -\alpha \pm j\beta$ and

  (4.1.1-8) $\hat{v}(s) = \frac{sv_{C0} - \frac{i_{L0}}{C}}{s^2 + 2\xi\omega_0 s + \omega_0^2} = \frac{(s+\alpha)D_1 + \beta D_2}{(s+\alpha)^2 + \beta^2}$

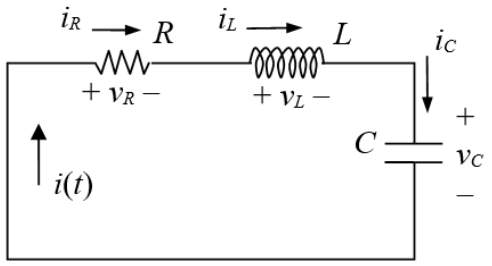
  $\Rightarrow v(t) = e^{-\alpha t}\left(D_1 \cos\beta t + D_2 \sin\beta t\right)$

  where $D_1 = v_{C0}$ and $D_2 = -\frac{\alpha v_{C0}}{\beta} - \frac{i_{L0}}{\beta C}$.

- From (4.1.1-1) and $v(t)$, we can obtain the currents through *R*, *L* and *C*.

<u>Example</u>: Consider a second-order parallel RLC circuit without any source. If $R = 4\ \Omega$, $L = 2$ H, and $C = 0.125$ F, determine $v(t)$ and $i_C(t)$ for $t \ge 0$ when $v(0) = 1$V and $i(0) = -1$A.

4.1.2 Series RLC circuit without any sources



- Component equations with initial conditions i_{L0} and v_{C0}:

(4.1.2-1) $v_R(t) = Ri(t)$, $v_L(t) = Li'(t)$, $v_C(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + v_{C0}$

- Mathematic model

(4.1.2-2) $i_R(t) = i_L(t) = i_C(t) = i(t)$

(4.1.2-3) $v_R(t) + v_L(t) + v_C(t) = Ri(t) + Li'(t) + \frac{1}{C}\int_0^t i(\tau)d\tau + v_{C0} = 0$

$$\Rightarrow R\hat{i}(s) + sL\hat{i}(s) - Li_{Lo} + \frac{1}{sC}\hat{i}(s) + \frac{v_{C0}}{s} = 0$$

Hence, we have

(4.1.2-4) $\hat{i}(s) = \dfrac{si_{Lo} - v_{C0}/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \dfrac{si_{Lo} - v_{C0}/L}{s^2 + 2\alpha s + \omega_0^2} = \dfrac{si_{Lo} - v_{C0}/L}{s^2 + 2\xi\omega_0 s + \omega_0^2}$

where $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$ and $\xi = \frac{\alpha}{\omega_0} = \frac{R}{2}\sqrt{\frac{C}{L}}$.

- Characteristic roots λ_1 and λ_2

The characteristic equation is $s^2 + 2\xi\omega_0 s + \omega_0^2 = (s - \lambda_1)(s - \lambda_2)$ and thus

(4.1.2-5) $\hat{i}(s) = \dfrac{si_{Lo} - \frac{v_{C0}}{L}}{s^2 + 2\xi\omega_0 s + \omega_0^2} = \dfrac{si_{Lo} - \frac{v_{C0}}{L}}{(s - \lambda_1)(s - \lambda_2)}$

where $\lambda_1, \lambda_2 = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)\omega_0$ are the characteristic roots.

- [A] $\xi = \frac{R}{2}\sqrt{\frac{C}{L}} > 1$

That means $\lambda_2 < \lambda_1 < 0$ and

(4.1.2-6) $\hat{i}(s) = \dfrac{si_{Lo} - \frac{v_{C0}}{L}}{(s - \lambda_1)(s - \lambda_2)} = \dfrac{A_1}{s - \lambda_1} + \dfrac{A_2}{s - \lambda_2}$

$$\Rightarrow i(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$

where $A_1 = \frac{\lambda_1}{\lambda_1 - \lambda_2} i_{L0} - \frac{1}{\lambda_1 - \lambda_2}\frac{v_{C0}}{L}$ and $A_2 = -\frac{\lambda_2}{\lambda_1 - \lambda_2} i_{L0} + \frac{1}{\lambda_1 - \lambda_2}\frac{v_{C0}}{L}$.

- [B] $\xi = \frac{R}{2}\sqrt{\frac{C}{L}} = 1$

  That means $\lambda_2 = \lambda_1 = -\xi\omega_0 = -\alpha$ and

  (4.1.2-7) $\hat{i}(s) = \frac{s i_{L0} - \frac{v_{C0}}{L}}{(s+\alpha)^2} = \frac{B_1}{s+\alpha} + \frac{B_2}{(s+\alpha)^2}$

  $\Rightarrow i(t) = (B_1 + B_2 t)e^{-\alpha t}$

  where $B_1 = i_{L0}$ and $B_2 = -i_{L0}\alpha - \frac{v_{C0}}{L}$.

- [C] $\xi = \frac{R}{2}\sqrt{\frac{C}{L}} < 1$

  That means $\lambda_1, \lambda_2 = \left(-\xi \pm j\sqrt{1-\xi^2}\right)\omega_0 = -\alpha \pm j\beta$ and

  (4.1.2-8) $\hat{i}(s) = \frac{s i_{L0} - \frac{v_{C0}}{L}}{s^2 + 2\xi\omega_0 s + \omega_0^2} = \frac{(s+\alpha)D_1 + \beta D_2}{(s+\alpha)^2 + \beta^2}$

  $\Rightarrow i(t) = e^{-\alpha t}\left(D_1 \cos\beta t + D_2 \sin\beta t\right)$

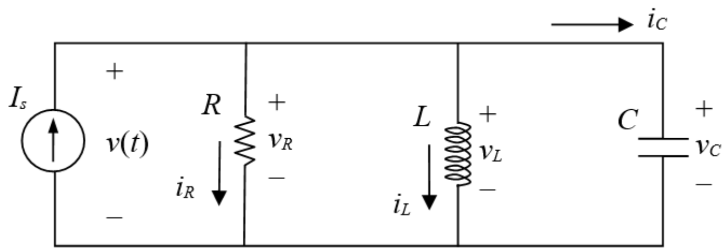
  where $D_1 = i_{L0}$ and $D_2 = -\frac{\alpha i_{L0}}{\beta} - \frac{v_{C0}}{\beta L}$.

- From (4.1.2-1) and $i(t)$, we can obtain the voltages across *R*, *L* and *C*.

<u>Example</u>: Consider a second-order series RLC circuit without any source. If $R = 4\ \Omega$, $L = 2$ H, and $C = 0.125$ F, determine $i(t)$ and $v_L(t)$ for $t \ge 0$ when $v(0) = 1\text{V}$ and $i(0) = -1\text{A}$.

4.2 Second-order RLC circuits with independent sources

4.2.1 Parallel RLC circuit with constant source



- Component equations with initial conditions i_{L0} and v_{C0}:

(4.2.1-1) $\quad i_R(t)=\dfrac{v(t)}{R}, \quad i_L(t)=\dfrac{1}{L}\int_0^t v(\tau)\,d\tau+i_{L0}, \quad i_C(t)=Cv'(t)$

- Mathematic model

(4.2.1-2) $\quad v_R(t)=v_L(t)=v_C(t)=v(t)$

(4.2.1-3) $\quad i_R(t)+i_L(t)+i_C(t)=\dfrac{v(t)}{R}+\dfrac{1}{L}\int_0^t v(\tau)\,d\tau+i_{L0}+Cv'(t)=I_s$

$$\Rightarrow \frac{\hat{v}(s)}{R}+\frac{1}{sL}\hat{v}(s)+\frac{i_{L0}}{s}+sC\hat{v}(s)-Cv_{C0}=\frac{I_s}{s}$$

Hence, we have

(4.2.1-4) $\quad \hat{v}(s)=\dfrac{sv_{C0}-\dfrac{i_{L0}-I_s}{C}}{s^2+\dfrac{s}{RC}+\dfrac{1}{LC}}=\dfrac{sv_{C0}-\dfrac{i_{L0}-I_s}{C}}{s^2+2\alpha s+\omega_0^2}=\dfrac{sv_{C0}-\dfrac{i_{L0}-I_s}{C}}{s^2+2\xi\omega_0 s+\omega_0^2}$

where $\alpha=\dfrac{1}{2RC}$, $\omega_0=\dfrac{1}{\sqrt{LC}}$ and $\xi=\dfrac{\alpha}{\omega_0}=\dfrac{1}{2R}\sqrt{\dfrac{L}{C}}$.

- Characteristic roots λ_1 and λ_2

The characteristic equation is $s^2+2\xi\omega_0 s+\omega_0^2=(s-\lambda_1)(s-\lambda_2)$ and thus

(4.2.1-5) $\quad \hat{v}(s)=\dfrac{sv_{C0}-\dfrac{i_{L0}-I_s}{C}}{s^2+2\xi\omega_0 s+\omega_0^2}=\dfrac{sv_{C0}-\dfrac{i_{L0}-I_s}{C}}{(s-\lambda_1)(s-\lambda_2)}$

where $\lambda_1,\lambda_2=\left(-\xi\pm\sqrt{\xi^2-1}\right)\omega_0$ are the characteristic roots.

- [A] $\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} > 1$

  That means $\lambda_2 < \lambda_1 < 0$ and

  (4.2.1-6) $\hat{v}(s) = \frac{sv_{C0} - \frac{i_{L0} - I_s}{C}}{(s-\lambda_1)(s-\lambda_2)} = \frac{A_1}{s-\lambda_1} + \frac{A_2}{s-\lambda_2}$

  $\Rightarrow v(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

  where $A_1 = \frac{\lambda_1 v_{C0}}{\lambda_1 - \lambda_2} - \frac{1}{\lambda_1 - \lambda_2}\frac{i_{L0} - I_s}{C}$ and $A_2 = -\frac{\lambda_2 v_{C0}}{\lambda_1 - \lambda_2} + \frac{1}{\lambda_1 - \lambda_2}\frac{i_{L0} - I_s}{C}$.

- [B] $\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} = 1$

  That means $\lambda_2 = \lambda_1 = -\xi\omega_0 = -\alpha$ and

  (4.2.1-7) $\hat{v}(s) = \frac{sv_{C0} - \frac{i_{L0} - I_s}{C}}{(s+\alpha)^2} = \frac{B_1}{s+\alpha} + \frac{B_2}{(s+\alpha)^2}$

  $\Rightarrow v(t) = (B_1 + B_2 t)e^{-\alpha t}$

  where $B_1 = v_{C0}$ and $B_2 = -v_{C0}\alpha - \frac{i_{L0} - I_s}{C}$.

- [C] $\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} < 1$

  That means $\lambda_1, \lambda_2 = \left(-\xi \pm j\sqrt{1-\xi^2}\right)\omega_0 = -\alpha \pm j\beta$ and

  (4.2.1-8) $\hat{v}(s) = \frac{sv_{C0} - \frac{i_{L0} - I_s}{C}}{s^2 + 2\xi\omega_0 s + \omega_0^2} = \frac{(s+\alpha)D_1 + \beta D_2}{(s+\alpha)^2 + \beta^2}$

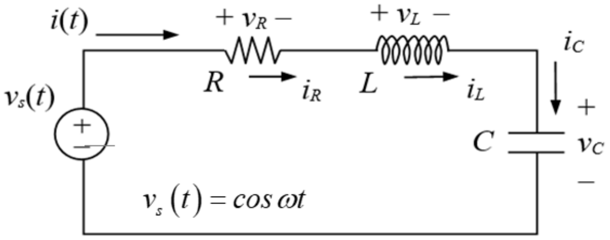
  $\Rightarrow v(t) = e^{-\alpha t}(D_1 \cos\beta t + D_2 \sin\beta t)$

  where $D_1 = v_{C0}$ and $D_2 = -\frac{\alpha v_{C0}}{\beta} - \frac{i_{L0} - I_s}{\beta C}$.

- From (4.2.1-1) and $v(t)$, we can obtain the currents through *R*, *L* and *C*.

Example: Consider a second-order parallel RLC circuit with independent current source $I_s = 2\,\text{A}$. If $R = 4\,\Omega$, $L = 2\,\text{H}$, and $C = 0.125\,\text{F}$, determine $v(t)$ and $i_C(t)$ for $t \geq 0$ when $v(0) = 1\text{V}$ and $i(0) = -1\text{A}$.

4.2.2 Series RLC circuit with voltage source



- Component equations with initial conditions i_{L0} and v_{C0}:

(4.2.2-1) $v_R(t)=Ri(t),\quad v_L(t)=Li'(t),\quad v_C(t)=\frac{1}{C}\int_0^t i(\tau)d\tau+v_{C0}$

- Mathematic model

(4.2.2-2) $i_R(t)=i_L(t)=i_C(t)=i(t)$

(4.2.2-3) $v_R(t)+v_L(t)+v_C(t)=Ri(t)+Li'(t)+\frac{1}{C}\int_0^t i(\tau)d\tau+v_{C0}=v_s(t)$

$$\Rightarrow R\hat{i}(s)+sL\hat{i}(s)-Li_{Lo}+\frac{1}{sC}\hat{i}(s)+\frac{v_{C0}}{s}=\frac{s}{s^2+\omega^2}$$

Hence, we have

(4.2.2-4) $\hat{i}(s)=\dfrac{s^3 i_{Lo}+si_{Lo}\omega^2+\frac{1-v_{C0}}{L}s^2-\frac{v_{C0}}{L}\omega^2}{\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)\left(s^2+\omega^2\right)}$

$$=\frac{\Psi(s)}{\left(s^2+2\alpha s+\omega_0^2\right)\left(s^2+\omega^2\right)}=\frac{\Psi(s)}{\left(s^2+2\xi\omega_0 s+\omega_0^2\right)\left(s^2+\omega^2\right)}$$

where $\Psi(s)=s^3 i_{Lo}+si_{Lo}\omega^2+\frac{1-v_{C0}}{L}s^2-\frac{v_{C0}}{L}\omega^2$, $\alpha=\frac{R}{2L}$, $\omega_0=\frac{1}{\sqrt{LC}}$ and $\xi=\frac{\alpha}{\omega_0}=\frac{R}{2}\sqrt{\frac{C}{L}}$.

- Characteristic roots λ_1, λ_2 and $\pm j\omega$

Let $\left(s^2+2\xi\omega_0 s+\omega_0^2\right)\left(s^2+\omega^2\right)=\left(s-\lambda_1\right)\left(s-\lambda_2\right)\left(s^2+\omega^2\right)$, then

(4.2.2-5) $\hat{i}(s)=\dfrac{\Psi(s)}{\left(s^2+2\xi\omega_0 s+\omega_0^2\right)\left(s^2+\omega^2\right)}=\dfrac{\Psi(s)}{\left(s-\lambda_1\right)\left(s-\lambda_2\right)\left(s^2+\omega^2\right)}$

where $\lambda_1,\lambda_2=\left(-\xi\pm\sqrt{\xi^2-1}\right)\omega_0$.

- [A] $\xi = \frac{R}{2}\sqrt{\frac{C}{L}} > 1$

That means $\lambda_2 < \lambda_1 < 0$ and

(4.2.2-6) $\hat{i}(s) = \frac{\Psi(s)}{(s-\lambda_1)(s-\lambda_2)(s^2+\omega^2)} = \frac{A_1}{s-\lambda_1} + \frac{A_2}{s-\lambda_2} + \frac{A_3 s + A_4\omega}{s^2+\omega^2}$

$$\Rightarrow\ i(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + A_3 \cos\omega t + A_4 \sin\omega t$$

where $A_1 = \frac{\Psi(\lambda_1)}{(\lambda_1-\lambda_2)(\lambda_1^2+\omega^2)}$, $A_2 = \frac{\Psi(\lambda_2)}{(\lambda_2-\lambda_1)(\lambda_2^2+\omega^2)}$,

$$A_3 - jA_4 = \frac{\Psi(j\omega)}{\omega^2(\lambda_1+\lambda_2) + j\omega(\lambda_1\lambda_2-\omega^2)}.$$

- [B] $\xi = \frac{R}{2}\sqrt{\frac{C}{L}} = 1$

That means $\lambda_2 = \lambda_1 = -\xi\omega_0 = -\alpha$ and

(4.2.2-7) $\hat{i}(s) = \frac{\Psi(s)}{(s+\alpha)^2(s^2+\omega^2)} = \frac{B_1}{s+\alpha} + \frac{B_2}{(s+\alpha)^2} + \frac{B_3 s + B_4\omega}{s^2+\omega^2}$

$$\Rightarrow\ i(t) = (B_1 + B_2 t)e^{-\alpha t} + B_3 \cos\omega t + B_4 \sin\omega t$$

where $B_2 = \frac{\psi(-\alpha)}{\alpha^2+\omega^2}$, $B_1 = \frac{\Psi'(-\alpha) - 2\alpha B_2}{\alpha^2+\omega^2}$,

$$B_3 - jB_4 = \frac{\Psi(j\omega)}{-2\alpha\omega^2 + j\omega(\alpha^2-\omega^2)}.$$

- [C] $\xi = \frac{R}{2}\sqrt{\frac{C}{L}} < 1$

That means $\lambda_1, \lambda_2 = \left(-\xi \pm j\sqrt{1-\xi^2}\right)\omega_0 = -\alpha \pm j\beta$ and

(4.2.2-8) $\hat{i}(s) = \frac{\psi(s)}{(s^2+2\xi\omega_0 s+\omega_0^2)(s^2+\omega^2)} = \frac{(s+\alpha)D_1 + \beta D_2}{(s+\alpha)^2+\beta^2} + \frac{sD_3 + \omega D_4}{s^2+\omega^2}$

$$\Rightarrow\ i(t) = e^{-\alpha t}(D_1 \cos\beta t + D_2 \sin\beta t) + D_3 \cos\omega t + D_4 \sin\omega t$$

where $D_1 - jD_2 = \frac{\psi(-\alpha + j\beta)}{2\alpha\beta^2 + j\beta(\alpha^2-\beta^2+\omega^2)}$,

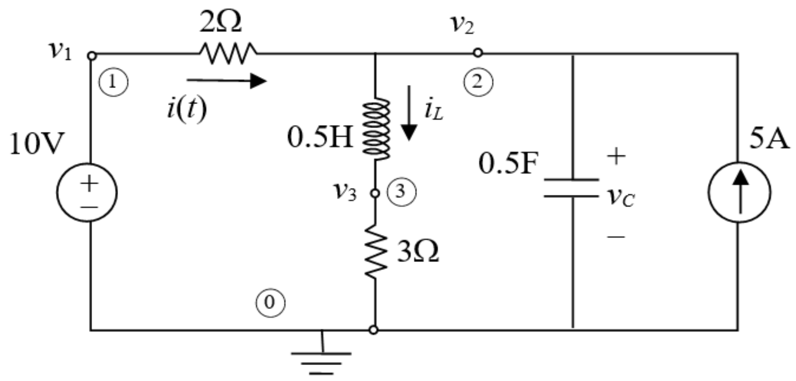
$$D_3 - jD_4 = \frac{\psi(j\omega)}{-2\alpha\omega^2 + j\omega(\alpha^2 + \beta^2 - \omega^2)}.$$

- From (4.2.2-1) and $i(t)$, we can obtain the currents through *R*, *L* and *C*.

Example: Consider a second-order series RLC circuit with voltage source $v_s(t) = sin\,2t$ V. If $R = 4\,\Omega$, $L = 2\,\text{H}$, and $C = 0.125\,\text{F}$, determine $i(t)$ and $v_L(t)$ for $t \geq 0$ when $v(0) = 1\text{V}$ and $i(0) = -1\text{A}$.

4.3 General linear RLC circuits

<u>Example</u>: Given $i_L(0) = 2\,\text{A}$ and $v_C(0) = 6\,\text{V}$, determine $i(t)$ for $t \geq 0$.



Node voltage equations:

KVL①: $v_1 = 10$

KCL②: $\frac{1}{2}(v_2 - v_1) + i_L + i_C - 5$

$$= \frac{1}{2}(v_2 - v_1) + \frac{1}{0.5}\int_0^t (v_2 - v_3)d\tau + 2 + 0.5v_2' - 5 = 0$$

KCL③: $-i_L + \frac{v_3}{3} = -\left(\frac{1}{0.5}\int_0^t (v_2 - v_3)d\tau + 2\right) + \frac{v_3}{3} = 0$

Taking Laplace transform yields

KVL①: $\hat{v}_1(s) = \frac{10}{s}$

KCL②: $0.5(\hat{v}_2(s) - \hat{v}_1(s)) + \frac{2}{s}(\hat{v}_2(s) - \hat{v}_3(s)) + \frac{2}{s}$

$$+0.5(s\hat{v}_2(s) - 6) - \frac{5}{s} = 0$$

KCL③: $-\left(\frac{2}{s}(\hat{v}_2(s) - \hat{v}_3(s)) + \frac{2}{s}\right) + \frac{\hat{v}_3(s)}{3} = 0$

i.e.,

(4.3-1) $\hat{v}_1(s) = \frac{10}{s}$

(4.3-2) $-s\hat{v}_1(s) + (s^2 + s + 4)\hat{v}_2(s) - 4\hat{v}_3(s) = 6s + 6$

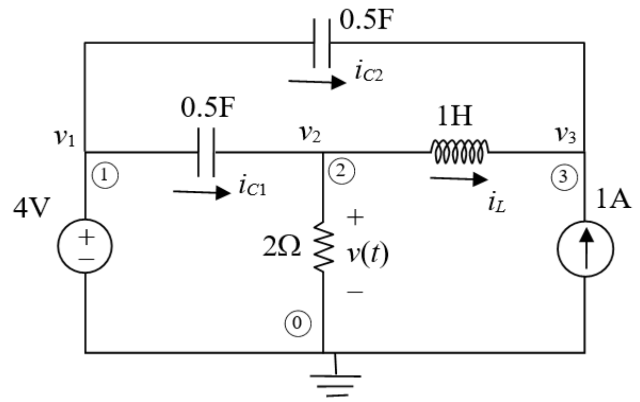
(4.3-3) $-6\hat{v}_2(s) + (s + 6)\hat{v}_3(s) = 6$

Hence, $\hat{v}_1(s)=\dfrac{10}{s}$, $\hat{v}_2(s)=\dfrac{6s^2+52s+120}{s(s^2+7s+10)}$, $\hat{v}_3(s)=\dfrac{6s^2+42s+120}{s(s^2+7s+10)}$

and $\hat{i}(s)=\dfrac{1}{2}(\hat{v}_1(s)-\hat{v}_2(s))=\dfrac{-1}{s}+\dfrac{10/3}{s+2}+\dfrac{-1/3}{s+5}$

The current is $i(t)=-1+\dfrac{10}{3}e^{-2t}-\dfrac{1}{3}e^{-5t}$, $t\geq 0$.

Example: If $i_L(0)=1\text{A}$ and there are no initial capacitor voltages, determine $v(t)$ for $t\geq 0$.



Node voltage equations:

KVL①: $v_1=4$

KCL②: $\dfrac{1}{2}v_2-i_{C1}+i_L=\dfrac{1}{2}v_2-0.5(v_1'-v_2')+\int_0^t(v_2-v_3)d\tau+1=0$

KCL③: $i_L+i_{C2}+1=\int_0^t(v_2-v_3)d\tau+1+0.5(v_1'-v_3')+1=0$

Taking Laplace transform yields

KVL①: $\hat{v}_1(s)=\dfrac{4}{s}$

KCL②: $\dfrac{1}{2}\hat{v}_2(s)-0.5s(\hat{v}_1(s)-\hat{v}_2(s))+\dfrac{1}{s}(\hat{v}_2(s)-\hat{v}_3(s))+\dfrac{1}{s}=0$

KCL③: $\dfrac{1}{s}(\hat{v}_2(s)-\hat{v}_3(s))+\dfrac{1}{s}+0.5s(\hat{v}_1(s)-\hat{v}_3(s))+\dfrac{1}{s}=0$

i.e.,

(4.3-1) $\hat{v}_1(s)=\dfrac{10}{s}$

(4.3-2) $s^2\hat{v}_1(s)-(s^2+s+2)\hat{v}_2(s)+2\hat{v}_3(s)=2$

(4.3-3) $s^2\hat{v}_1(s)+2\hat{v}_2(s)-(s^2+2)\hat{v}_3(s)=-4$

It can be obtained that $\hat{v}_2(s)=\dfrac{10s^3-2s^2+40s+4}{s(s^3+s^2+4s+2)}$.

Since $v(t)=v_2(t)$, we have

(4.3-4) $$\hat{v}(s)=\frac{10s^3-2s^2+40s+4}{s(s^3+s^2+4s+2)}$$

$$=\frac{A}{s}+\frac{B}{s+0.5332}+\frac{(s+0.2334)C+1.9227D}{(s+0.2334)^2+1.9227^2}$$

Once A, B, C and D are solved, the voltage is expressed as

(4.3-5) $v(t)=A+Be^{-0.5332t}+e^{-0.2334t}(C\,cos1.9227t+D\,sin1.9227t)$.